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# LIMB VOLUME ESTIMATES BASED ON LIMB ELLIPTICAL VS. CIRCULAR CROSS SECTION MODELS

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*ABSTRACT*

*Limb volumes, as would be estimated by the widely used right circular truncated cone model (right circular frustum), were analytically compared to volume estimates that would be obtained if limbs were represented by an elliptical cross section. A general expression for the ratio of circular to elliptical limb segmental volumes was developed in terms of the ratio of minimum to maximum limb radial dimensions. Analytical results showed that in general the elliptical representation resulted in smaller calculated limb volumes, with the difference increasing as the ratio of minimum to maximum limb dimension became smaller. However, differences in estimated limb volume between circular and elliptic representations were less than 5% if the minimum to maximum limb dimensions at measured circumference sites were greater than 0.64. It is concluded that although limbs deviate from circularity, the added work of determining minimum and maximum dimensions for each circumference measured, as is needed to employ elliptical models, is warranted only for extreme differences in limb radial dimensions or possibly for research purposes.*

Measurement of limb volume and its change through a course of lymphedema therapy is most often clinically accomplished via tape measure circumference measurements which then are incorporated into formulae that estimate limb volumes based on an assumed geometrical model of the limb (1). A widely used formula is based on the right circular truncated cone model, also called the frustum model. Experimental work in which volume estimates of this model were compared with volumes measured by water displacement (2) and optoelectronic methods (3, 4) have generally shown good correlation although systematic differences in absolute volume determined by water displacement and circumference measurements have been described (5). An intrinsic assumption of the frustum model is that the cross section of the limb is circular at each measured circumference site, an assumption that is often not true in normal or swollen limbs. However, the effect of deviations from circularity on calculated limb volume using the frustum model has not been reported. Thus, the focused goal of this brief report was to determine the effect of deviations from circularity on estimated limb volume.

## *METHODS*

### *Elliptical Frustum Model*

The limb cross sectional shape at two potential circumference measurement sites separated by distance L was taken as being elliptical. The major and minor dimensions were denoted A and B for the one section and as a and b for the other section *(Fig. 1)*. The

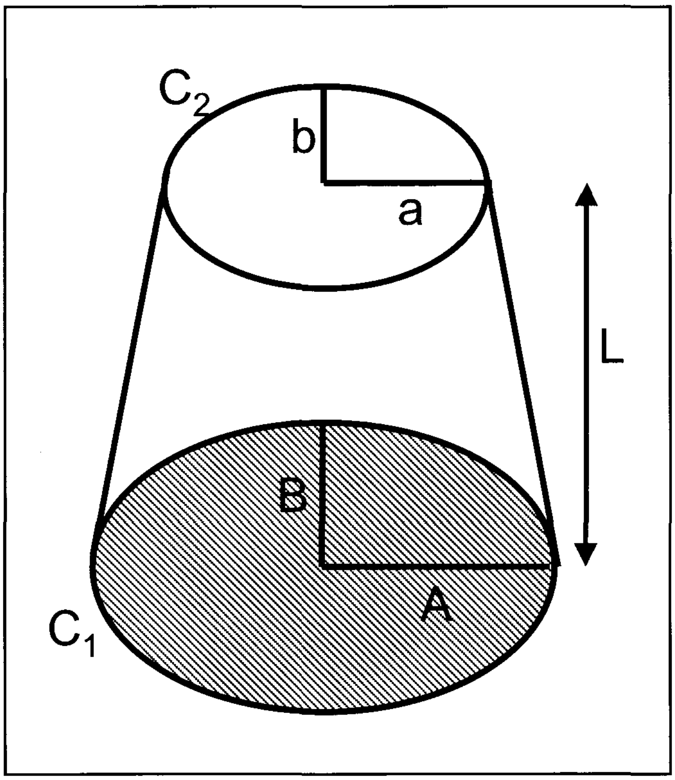


*Fig. 1. Elliptic frustum representation of limb segment. $C_1$ and $C_2$ are circumferences of two sections separated by a distance L with the larger section having major and minor dimensions A and B and the smaller segment having major and minor dimensions a and b.*

segmental volume included within the segments is determined as the volume of the resultant right elliptical frustum. Using the method of similar triangles it can be shown that the segmental volume ($V_E$) is given by the general relationship

[1] $V_E = (\pi L/3)(A^2B - a^2b) / (A - a)$

The well known formula for volumes of circular cross section frustums arises as a special case of this general relationship by letting A=B=R and a=b=r in which R and r are the radii of the two circular cross sections. This results in the relationship for the volume of circular frustum ($V_C$) given as

[2] $V_C = (\pi L/12)(D^3 - d^3) / (D - d)$

in which D and d are the diameters of the respective circular sections.

From relationships [1] and [2], the ratio of frustum volumes for the circular and elliptical cross sectional shapes can be expressed as

[3] $V_C/V_E = (1/4)[(D^3 - d^3) / (D - d)] / [(A^2B - a^2b) / (A - a)]$

*Volume Comparisons*

An appropriate comparison of volume predictions between elliptic and circular models requires that the comparison be made on the basis of equal measured circumferences. Thus, the central question could be stated as follows. For any given measured

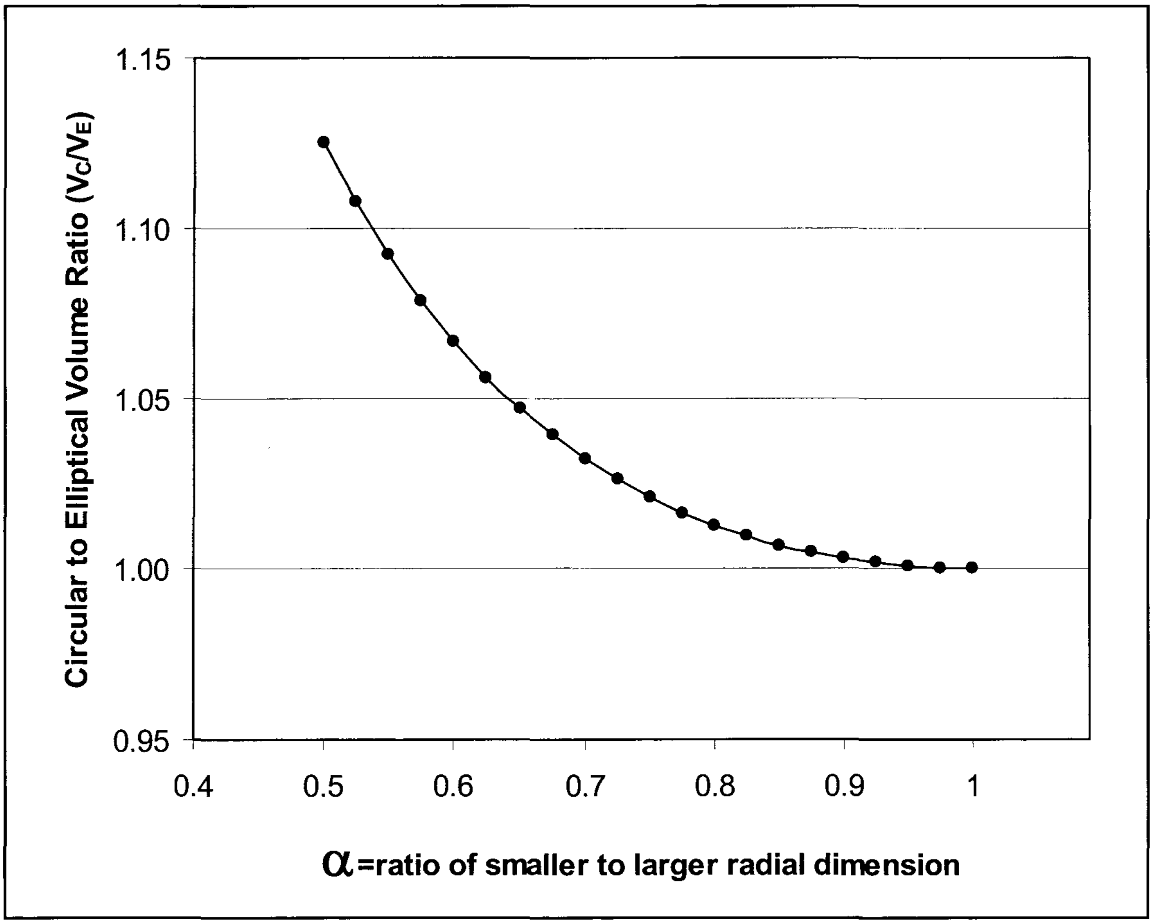


*Fig. 2. Volume ratios determined for circular to elliptical limb models. For the same measured circumferences, the circular model always yields a larger calculated volume. Alpha (_) is the ratio of minor to major limb radial dimension at each measured circumference.*

limb circumference pair ($C_1$ and $C_2$), what segmental volume difference would be expected if the cross section was elliptical vs. circular? To determine this, the circumference of an elliptical shape needs to be expressed in terms of its major and minor axis dimensions. The exact formulation of this relationship is given by the solution of a complete elliptic integral of the 2nd kind. However, to an excellent approximation, the circumference C can be expressed by the first term of a rapidly converging infinite series (6) as

[4] $C_1 = \pi(A + B)$ and $C_2 = \pi(a + b)$

To compare volumes on the basis of equal circumferences, these relationships show that the diameters D and d of the circular cross sections should be expressed as A+B and a + b, respectively. When this is done one gets as the most general relationship for the volume ratios

[5] $V_C/V_E = (1/4)\ \{[(A+B)^3 - (a+b)^3] / [(A+B) - (a+b)]\} / [(A^2B - a^2b) / (A - a)]$

To simplify, we take ratio of the smaller elliptical dimension to the larger to be equal to $\alpha$ such that $B=\alpha A$ and $b=\alpha a$ ($\alpha<=1$).

When substituted into [5], the final volume ratio, circular to elliptical, takes the following compact form.

[6] $V_C/V_E = (1/4)(1+\alpha)^3/\alpha(1+\alpha)$

*RESULTS*

A circular cross section is defined in equation [6] if $\alpha = 1$. For this case the ratio $V_C/V_E$ is 1 as expected. For all other cases ($\alpha < 1$) the volume ratio is > 1 indicating that the circular frustum model yields a larger volume than does the elliptical frustum model. However, except for extreme differences in ellipse major and minor dimensions (small $\alpha$), the difference between the two volumes is not large. This is shown graphically for an $\alpha$ range of 1 to 0.5 in *Fig. 2.* From these analyses, circular volumes exceed elliptical volumes by 5%, 10%, 15% or 20% if $\alpha$ equals 0.64, 0.54, 0.47 or 0.42, respectively.

*DISCUSSION*

The main results show that volume estimates using frustum formulas to estimate limb volume are little affected by the choice of either circular or elliptical models for the limb unless there is a substantial difference in the ratio of smaller to larger limb radial dimensions. Thus, although limbs deviate from circularity, the added work of determining minimum and maximum dimensions for each circumference measured, as is needed to employ elliptical models, is warranted only in extreme conditions or possibly for research purposes.

*REFERENCES*

1. Casley-Smith, JR: Measuring and representing peripheral oedema and its alterations. Lymphology 27 (1994), 56-70.
2. Pani, SP, P Vanamail, J Yuvaraj: Limb circumference measurement for recording edema volume in patients with filarial lymphedema. Lymphology 28 (1995), 57-63.
3. Stanton, AW, JW Northfield, B Holroyd, et al: Validation of an optoelectronic limb volumeter (Perometer). Lymphology 30 (1997), 77-97.
4. Mayrovitz, H, N Sims, J Macdonald: Limb volume measurement in patients with edema. Adv. Skin Wound Care. 113 (2000), 272-276.
5. Sander, AP, NM Hajer, K Hemenway, et al: Upper-extremity volume measurements in women with lymphedema: A comparison of measurements obtained via water displacement with geometrically determined volume. Phys. Ther. 82 (2002), 1201-1212.
6. Ramanujan, K: Modular equations and approximations to PI. Q. J. Pure Applied Math. 45 (1913), 350-372.

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